# DIURNAL CYCLES OF THE REFRACTIVE INDEX STRUCTURE FUNCTION COEFFICIENT

Marvin L. Wesley, et al

Ballistic Research Laboratories

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by

Marvin L. Wesely
Ernest C. Alcaraz



November 1972



USA BALLISTIC RESEARCH LABORATORIES
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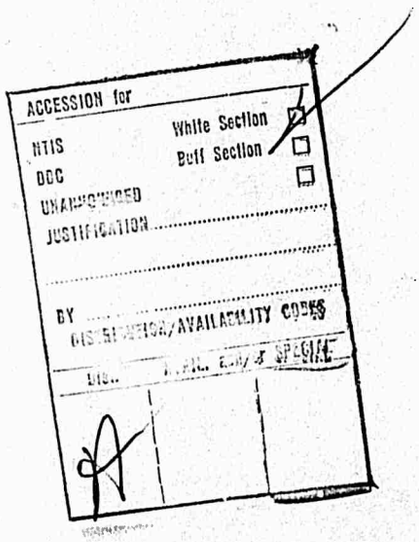


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13. ABSTRACT

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# DIURNAL CYCLES OF THE REFRACTIVE INDEX STRUCTURE FUNCTION COEFFICIENT

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## ABSTRACT

The refractive index structure function coefficient, $C_n^2$, is an atmospheric parameter needed to describe scintillation and small-scale phase fluctuations of electromagnetic radiation propagated in the atmospheric surface layer. Since systematic direct measurements of $C_n$ for many climates and seasons are not available, an indirect method is developed where $C_n$ is calculated from the estimates of sensible and latent heat flux components of the surface energy budgets. This indirect method is primarily for heights less than 4 meters, because low intermittency and $K_H/K_M \simeq 1$ are assumed. Diurnal variations of $C_n$ at several heights above land for six combinations of climates, seasons, and surface conditions are calculated from heat fluxes measured by different investigators at many locations, for moderate to high wind velocities. These predictions of $C_n$ agree well with some direct measurements of $C_n$ when assumptions of nearly ideal weather and sites are met. The effect of water vapor on $C_n$ is usually a reduction of about $3/\beta$ per cent and thus is usually negligible above land, but can be significant above tropical oceans.

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## INTRODUCTION

For optical radiation the atmospheric index of refraction variations primarily result from temperature-induced density changes. These variations cause scintillation and phase fluctuations of laser beams propagated in the atmosphere and the loss of resolution of objects viewed through high-magnification telescopes. Temperature variations also help cause the scattering of acoustic and radar signals, phenomena used in the remote sensing of the atmosphere. The length scales of the temperature inhomogeneities that distort image and laser beam propagation in the atmospheric surface layer are usually on the order of centimeters for scintillation and tens of centimeters for phase fluctuations, and the eddies with these sizes are in the inertial subrange where fluctuations are isotropic. Isotropy greatly aids the development of theories like those in *Tatarski* [1961] that describe electromagnetic propagation, and only one atmospheric parameter, the refractive index structure function coefficient, $C_n^2$, is needed. Using Kolmogorov's theory one can determine $C_n$ for optical radiation from a variance of temperature differences between two fast-response thermometers or from a temperature power spectrum at one thermometer. When the ideal case of the temperature and wind structure being horizontally homogeneous in the surface layer exists, one measurement suffices and $C_n$ can be inferred for any height within a few meters of the surface.

The purpose of this paper is to present diurnal variations of $C_n$ for optical radiation at several heights in the atmospheric surface layer for various climates, seasons, and surface characteristics. However, since direct systematic measurements of $C_n$ are not available, an indirect method is developed, compared with similar indirect methods, and used to obtain diurnal trends. The indirect method uses sensible and latent heat flux components of the surface energy budgets, which have become abundantly available in the meteorological and climatological literature. Representative wind velocities are chosen and the effect of water vapor is evaluated. Also, the model developed

here is for use primarily at heights less than 4 meters, since the effect of intermittency at greater height limits the validity of necessary empirical functions.

Some of the terminology used in this paper must be explained. First, "turbulence" is not synonymous with "temperature fluctuations" as is common in atmospheric optics [*Lawrence et al.*, 1970]. It will be shown that water vapor fluctuations can be important relative to temperature fluctuations above a lake or an ocean. Also, wind velocity fluctuations can significantly affect the propagation of high-power laser beams. Thus "turbulence" implies any fluctuating property of the air, and equating a value of $C_n$ with intensity of turbulence is misleading unless thermal turbulence is specified. Second, "stable" conditions refer to an inversion or an increase of potential temperature with height, and heat being convected from the air to the cooled ground. "Neutral" conditions are adiabatic conditions; $C_n$ approaches zero and the average heat flux is zero. "Unstable" conditions are lapse conditions, or a decrease beyond the normal adiabatic decrease of temperature with height, and heat is being convected from the surface to the air.

## INDIRECT CALCULATION OF $C_n$

The structure function, $D_T$, for temperature fluctuations is given by

$$D_T = \langle (T_1 - T_2)^2 \rangle , \qquad (1)$$

where $T_1$ and $T_2$ are the temperatures at two points fixed on a line oriented normally to the mean wind direction and separated by distance r. Two thermometers can be used to measure $T_1$ and $T_2$, but an alternative method is to use one thermometer and determine $T_2$ by lagging in time from $T_1$ by $\Delta t$. Then Taylor's Hypothesis of a frozen field is assumed and $r = \Delta t\ \bar{u}$, where $\bar{u}$ is the mean longitudinal wind speed [*Lumley and Panofsky*, 1964]. When r is of the order of inertial

subrange scales, Kolmogorov's reasoning can be applied [*Obukhov*, 1949; *Corrsin*, 1951] to yield the following:

$$D_T = C_T^2 r^{2/3}, \tag{2}$$

where the coefficient is

$$C_T^2 = AN\varepsilon^{-1/3}. \tag{3}$$

The value of A is constant, N is the rate of dissipation of temperature fluctuations, and $\varepsilon$ is the rate of dissipation of mechanical turbulent energy.

*Tatarski* [1961] suggests that in a homogeneous, statistically stationary, atmospheric surface layer the rate of dissipation of temperature fluctuations is equal to the product of the vertical heat flux, H, and the mean vertical temperature gradient, $\partial\overline{T}/\partial z$. Data given by *Wyngaard et al.* [1971] indicate that this is a good assumption.

$$N = H \frac{\partial \overline{T}}{\partial z} (\rho C_p) \tag{4}$$

where $(\rho C_p)$ is the heat capacity of air and $H(\rho C_p) = \langle w'T' \rangle$ is the covariance of the vertical wind and temperature (the primes indicate deivations from the means). In the atmospheric surface layer, $\varepsilon$ is the sum of mechanical and buoyant production rates of mechanical and turbulent energy [*Lumley and Panofsky*, 1964].

$$\varepsilon = u^{*2} \frac{\partial \overline{u}}{\partial z} (1 - Rf) \tag{5}$$

where $u^* = \langle u'w' \rangle^{1/2}$ is the surface friction velocity, $\overline{u}$ is the mean longitudinal wind speed, and Rf is the flux Richardson number. In this paper, terms that may cause equation 5 to be in error by up to 40 percent for non-neutral conditions [*Record and Cramer*, 1966; *Zubkovskiy and Koprov*, 1970; *Wyngaard and Coté*, 1970; *McBean et al.*, 1971] are neglected because of disagreement about the sign and magnitude of the error terms. The value of A in equation 3 is obtained from temperature spectrum analyses and a representative value of 2.8 is used here [*Panofsky*, 1969 *Pond et al.*, 1971; *Wyngaard and Coté*, 1971]. This value is very close

to the constant for the transverse wind structure function coefficient, but no theory states these constants must be the same. Since temperature is a scalar, there is no longitudinal form of equation 2 as exists for wind components, where the longitudinal constant is 3/4 the transverse constant.

Substituting equations 4 and 5 into equation 3, we find

$$C_T^2 = 2.8 \langle w'T' \rangle \frac{\partial \overline{T}}{\partial z} \left[ u^{*2} \frac{\partial \overline{u}}{\partial z} (1\text{-}Rf) \right]^{-1/3} . \qquad (6)$$

*Lumley and Panofsky* [1964] list several well-known relationships we can use to simplify equation 6.

$$\langle u'w' \rangle = K_M \frac{\partial \overline{u}}{\partial z} , \qquad (7a)$$

$$\langle w'T' \rangle = K_H \frac{\partial \overline{T}}{\partial z} , \qquad (7b)$$

where $K_M$ and $K_H$ are the eddy diffusivities for momentum and heat, respectively. The average wind speed and temperature gradients are

$$\frac{\partial \overline{u}}{\partial z} = \frac{u^* \phi_M}{kz} , \qquad (8a)$$

$$\frac{\partial \overline{T}}{\partial z} = \frac{T^* \phi_H}{kz} , \qquad (8b)$$

where $k = 0.4$ is von Karmon's constant, $T^* = \langle w'T' \rangle / u^*$, and $\phi_M$ and $\phi_H$ are the nondimensional shears of wind and temperature, respectively. Substitutions of equations 7 and 8 into equation 6 result in these final equations:

$$C_T z^{1/3} / T^* = 1.67 \phi_M^{1/2} \left[ \alpha^{1/2} k^{1/3} (1\text{-}Rf)^{1/6} \right]^{-1} \qquad (9)$$

and

$$C_T / \left( \frac{\partial \overline{T}}{\partial z} z^{2/3} \right) = 1.67 k^{2/3} \alpha^{1/2} \left[ \phi_M^{2/3} (1\text{-}Rf)^{1/6} \right]^{-1} , \qquad (10)$$

where $\alpha = K_H/K_M = \phi_M/\phi_H$ .

Atmospheric variables can be nondimensionalized with u* and T* since they do not vary with height in the atmospheric surface layer. All properly nondimensionalized parameters can be characterized in terms of z/L where L is the Monin-Obukhov length.

$$z/L = zkgT^*(\overline{T}u^{*2})^{-1}, \tag{11}$$

where g is the acceleration of gravity. The right-hand sides of equations 9 and 10 have three parameters that must be evaluated as a function of z/L. The flux Richardson number is an explicit function of z/L: $Rf = \phi_M^{-1} z/L$. The function $\phi_M$ has been established by the empirical KEYPS function, but for computation purposes we choose to use the following:

$$\phi_M = 1+4.5(z/L) \qquad (z/L > 0), \tag{12a}$$

$$\phi_M = 1+4.5(z/L)\exp(4.5z/L) \qquad (0.> z/L \geq -0.04), \tag{12b}$$

$$\phi_M = 0.08+0.285(-z/L)^{1/3} \qquad (z/L < -0.04). \tag{12c}$$

These equations result in curves of $\phi_M$ verses z/L that closely agree with those given by *Businger et al.* [1971]. For $\alpha$, however, widely different estimates have been obtained [*Laykhtman and Ponomarev*, 1969; *Businger et al.*, 1971]. Here we will base our estimates on measurements described by *Wesely et al.* [1970] which were within 4 meters of the surface. The value of $\alpha$ was apparently near unity for all stability conditions since flux estimates from aerodynamic methods using $\alpha = 1$ agreed with direct flux estimates via eddy correlation techniques. The following forms for $\alpha$ are assumed:

$$\alpha = (1+2.25z/L) \qquad (z/L > 0), \tag{13a}$$

$$\alpha = 1-0.225z/L \qquad (z/L \leq 0). \tag{13b}$$

At heights greater than 4 meters, equation 13 is inaccurate since, as shown by *Haugen and Kaimal* [1971], intermittency increases with height and calculations by *Stearns* [1971] show that with highly intermittent conditions $\alpha$ becomes much larger than unity.

After substitution of equations 12 and 13 into equation 9, the curves in Fig. 1 can be calculated. Also shown is a curve determined from data collected at $z \geq 5.66$ meters, as summarized by *Wyngaard et al.* [1971]. The two curves differ because of the higher level of intermittency that existed at the greater height, thus increasing $\alpha$ or, alternatively, increasing T* with respect to $C_T$ in equation 9.

The value of $C_T$ can also be determined from the mean gradients of temperature and horizontal wind rather than the fluxes of heat and momentum. This method uses the gradient Richardson number, Rg, as the stability parameter: $Rg = \phi_M^{-1}\alpha^{-1}z/L$. Substituting equations 12 and 13 into equation 10 results in Fig. 2, which also shows results from the same experiment cited for Fig. 1 ($z/L \geq 5.66$ meters). The effect of the increased level of intermittency at $z > 4$ meters is to increase the ordinate values because $\alpha$ is increased or, alternatively, $\frac{\partial \overline{T}}{\partial z}$ is decreased with respect to $C_T$ in equation 10. The remaining curve in Fig. 2 was determined by *Tsvang* [1960] from spectral analysis of temperature fluctuation measured at heights of 1 and 4 meters. Tsvang's curve appears low for unstable conditions ($Rg < 0$), as is also indicated by his comparisons with previous Russian estimates obtained with equation 1. Experimental data by *Livingston et al.* [1970] have shown that Tsvang's curve is reasonably accurate.

If only temperature changes significantly affect the air density and thus its refractive index,

$$C_n^2 = \left[A_1 p / \overline{T}^2\right]^2 C_T^2 , \qquad (14)$$

where p is the static atmospheric pressure and $A_1$ is a constant for a particular radiation wavelength. However, fluctuations in water vapor

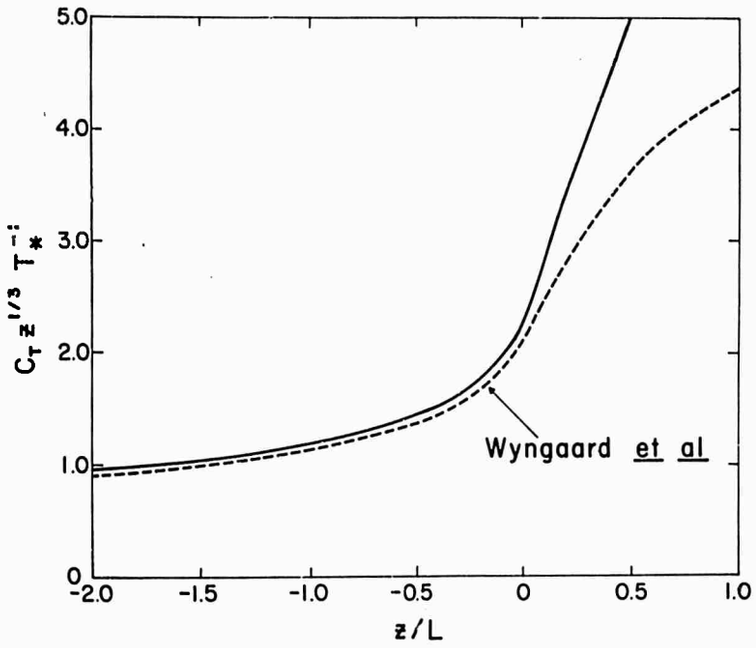


Figure 1 - Dependence of $C_T$ on z/L. The solid line was computed from equation 9.

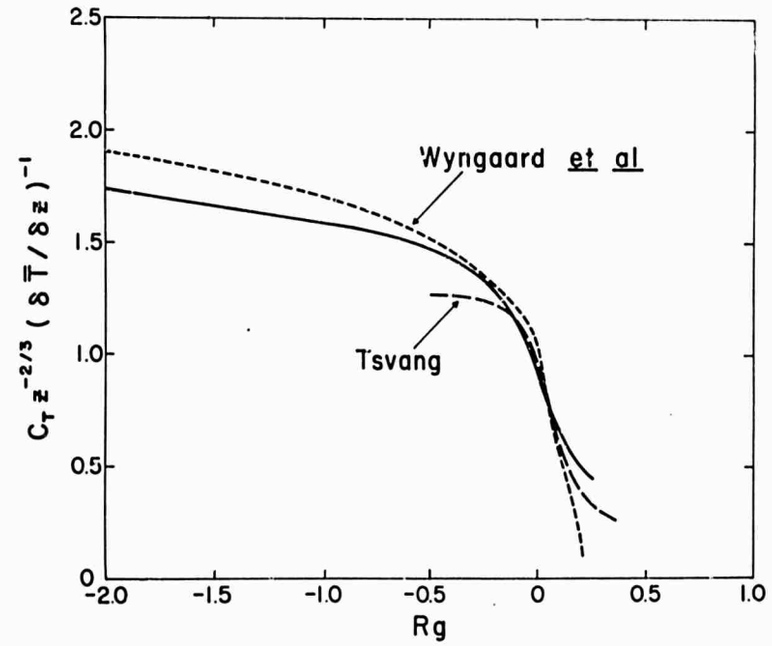


Figure 2 - $C_T$ nondimensionalized with the local mean temperature gradient plotted against Rg. The solid line was computed from equation 10.

concentration can also affect air density. To evaluate this effect, we assume that the eddy diffusivities for scalar quantities are identical at any given time and location. This is justified by the success [e.g., *Wesely et al.*, 1970] of equating $K_H$ with $K_W$, the eddy diffusivity for water vapor, in the Bowen's ratio method of computing vertical sensible and latent heat flux in the atmospheric surface layer [*Lumley and Panofsky*, 1964]. With replacement of T by n in equations 7b and 6 and the assumption of $K_n = K_H$, it can be shown that

$$C_n = C_T \langle w'n' \rangle / \langle w'T' \rangle \tag{15}$$

The atmospheric refractive index is given by *Fleagle* [1950] as

$$n - 1 = A_1 p / T + A_2 e/T,$$

where e is the water vapor pressure and $A_2$ is a constant for a particular radiation wavelength. Both $A_1$ and $A_2$ can be evaluated with the Barrell and Sears formula [*List*, 1958] using the technique described by *Kallistratova and Timanovsky* [1971]; $A_1 = 78.7 \times 10^{-6}$ °K $mb^{-1}$ and $A_2 = 12.4 \times 10^{-6}$ °K $mb^{-1}$ for a wavelength of 0.6μ. The value of $A_1$ varies only a few percent over the visible and near infrared spectrum, but $A_2$ varies greatly for different infrared wavelengths because of anomalous dispersion [*Chalfee*, 1968].

When pressure fluctuations and third order correlations are neglected, it can be shown that for visible light

$$\langle w'n' \rangle = -A_1 p \overline{T}^{-2} \langle w'T' \rangle \left[1 - \overline{T} A_2 C_p (0.622 A_1 V\beta)^{-1}\right] . \tag{16}$$

Therefore, at $\overline{T} = 288$°K and p = 1013 mb,

$$C_n = C_T A_1 p \overline{T}^{-2} \left[1 - 0.0298\beta^{-1}\right], \tag{17}$$

where V is the latent heat of vaporization and $\beta$ is Bowen's ratio. Since $\beta = H/E$ where E is the latent heat flux, a high value of $\beta$ in the daytime usually means that energy used in evaporation is much smaller than the thermal convective loss from the heated surface. This is the most frequent case above land since $\beta > 0.2$ is typical, and the correction in equation 17 is negligible there. Usually $\beta > 0$ because the earth's surfaces are most often either warming and evaporating or cooling and condensing; air parcels warmer than the surrounding air usually have a higher specific humidity. However, when surface cooling and evaporation occur simultaneously, $\beta < 0$ can be found above oceans and lakes when severe advection occurs and occasionally in a dry atmosphere above moist ground at night.

## DIURNAL CYCLES OF $C_n$

Diurnal trends of $C_n$ are shown in Figs. 3 through 8 for different micrometeorological conditions above land at middle latitudes. Hundreds of typical trends can occur, but a particular micrometeorological situation on a clear and cloudless day should yield either one of the diurnal trends shown here or something between two of the graphs. The variables that determine which curve to use are listed in Table 1, and they can be summarized as follows: (1) season and latitude, which determine the diurnal solar radiation cycle; (2) the amount of water available for evapotranspiration from the surface, whose main effect is to reduce the surface temperature rise; and (3) the radiation absorption characteristics of the surface, usually determined by the type and condition of vegetation, which is strongly dependent upon the above two factors and man's modifications. Another factor that must be taken into account is that $C_n$ is approximately proportional to $1/u^*$. In Figs. 3 through 8, moderate wind speeds of about 3 m/sec at a 2-meter height ($u^* = 25$ m/sec) are assumed when $H < 125$ W $m^{-2}$, and strong wind speeds of about 5 m/sec at a 2-meter height ($u^* = 40$ m/sec) are assumed when $H \geq 125$ W $m^{-2}$.

TABLE 1. Assumed climatic conditions for $C_n$ estimates in Figs. 3-8.

| | Possible Climates | | | | Soil Moisture | | |
|---|---|---|---|---|---|---|---|
| Case | Rainfall[a] | Latitude | Example | Season | Subsurface | Surface | Surface |
| 1 | >25 | 30°-50° | Treeless area | Winter | Wet[b] | Wet[b] | Snow |
| 2 | >25 | 30°-50° | Treeless area | Winter | Wet[b] | Wet[b] | Short brown grass |
| 3 | 75-125 | 30°-50° | Wet grasslands | Summer | Wet | Wet | Short green grass |
| 4 | 50-100 | 30°-50° | Plains, steppes | Summer | Wet | Dry | Short green grass |
| 5 | <25 | 10°-40° | Desert | Winter | Dry | Dry | Bare sand, sparse or no plants |
| 6 | <25 | 10°-40° | Desert | Summer | Dry | Dry | |

[a] cm/year
[b] frozen

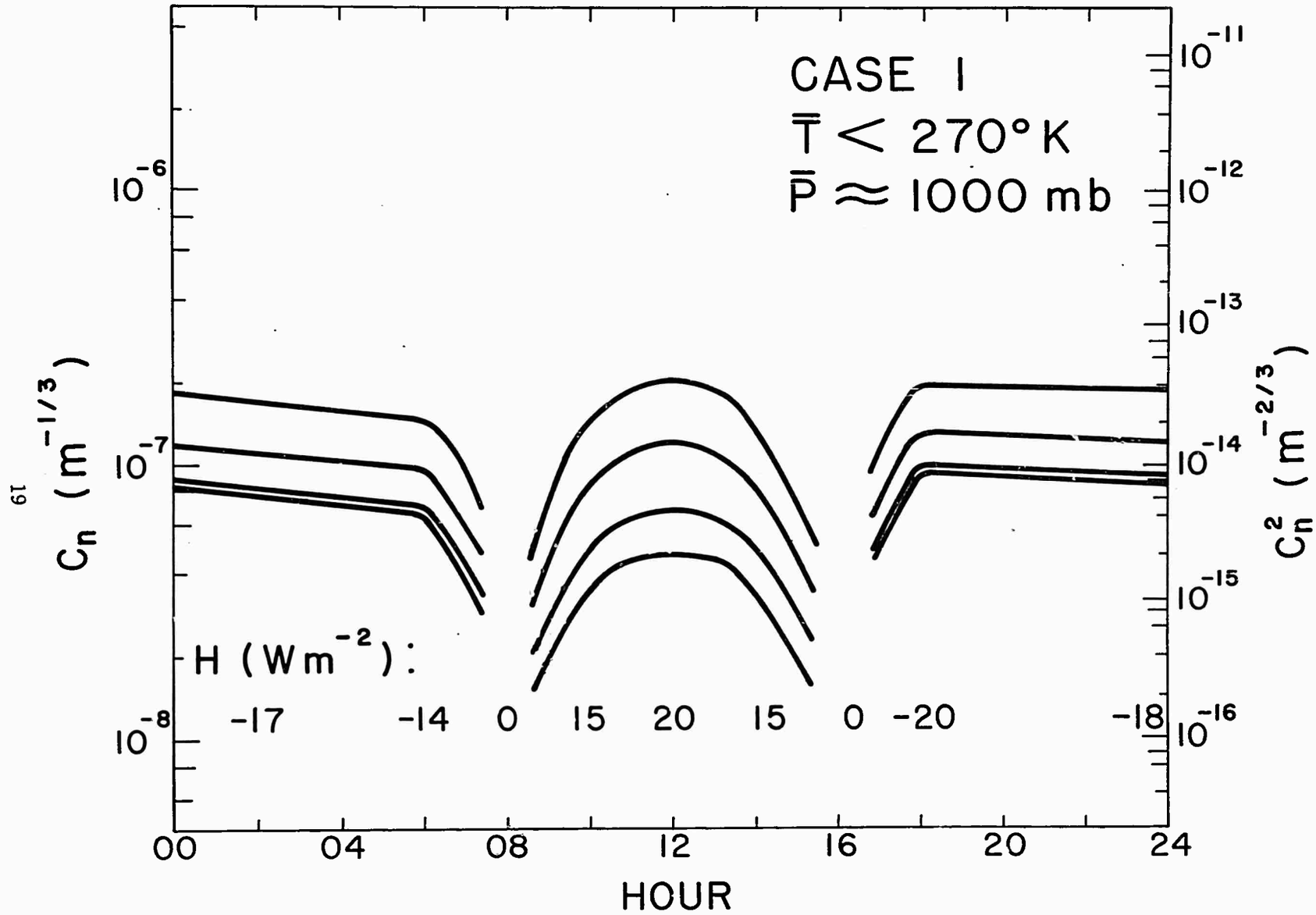


Figure 3 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 1.

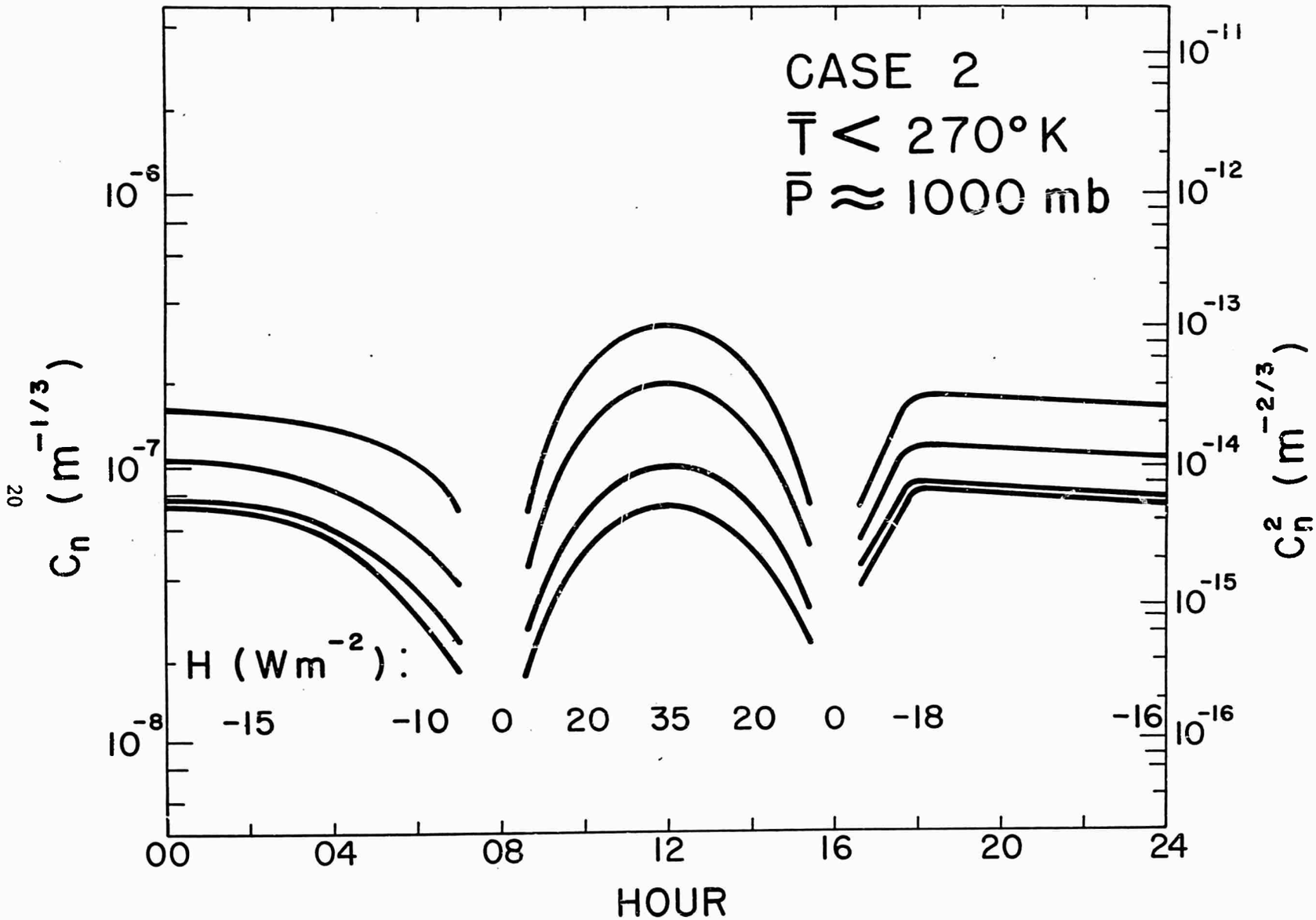


Figure 4 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 2.

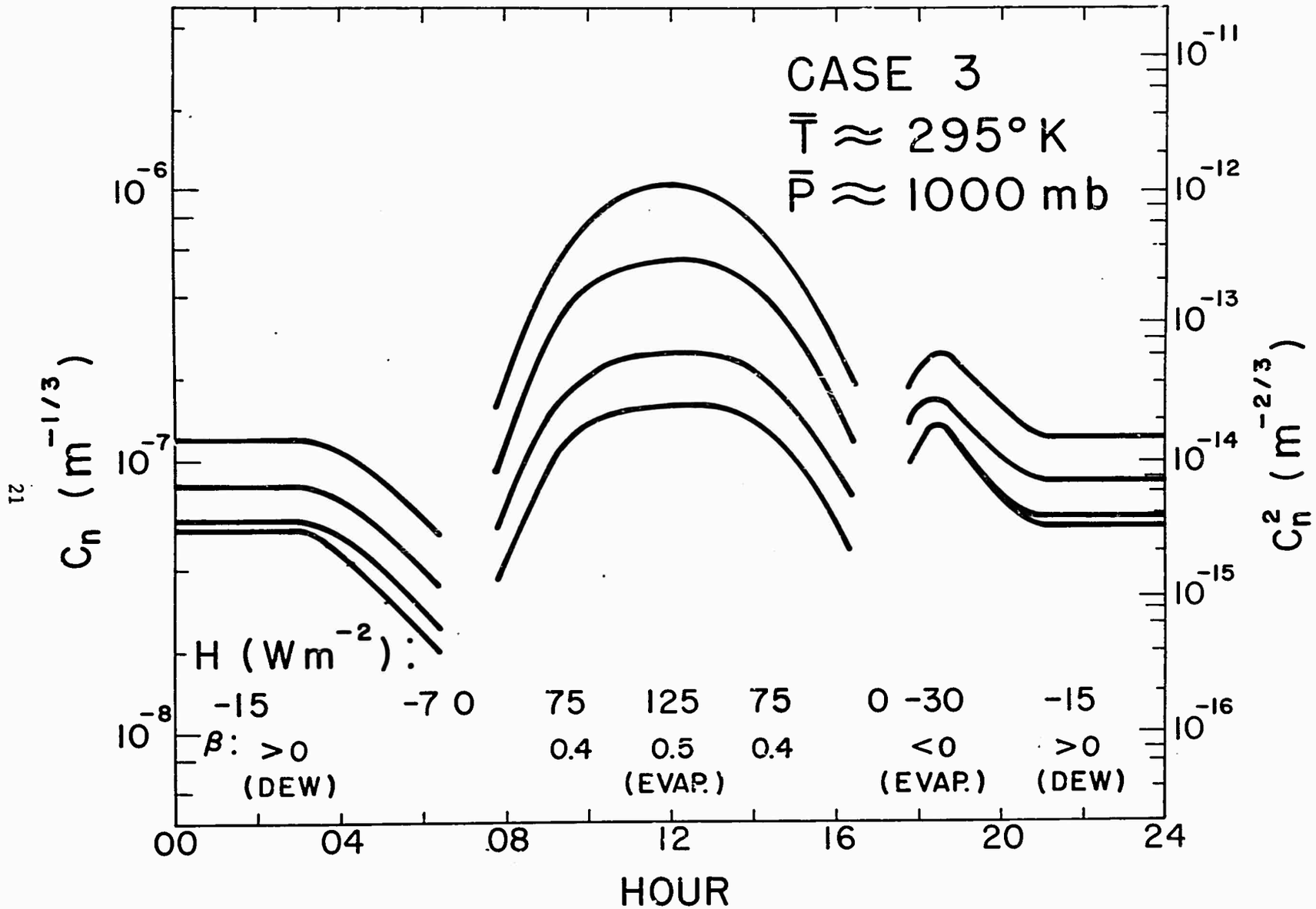


Figure 5 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 3.

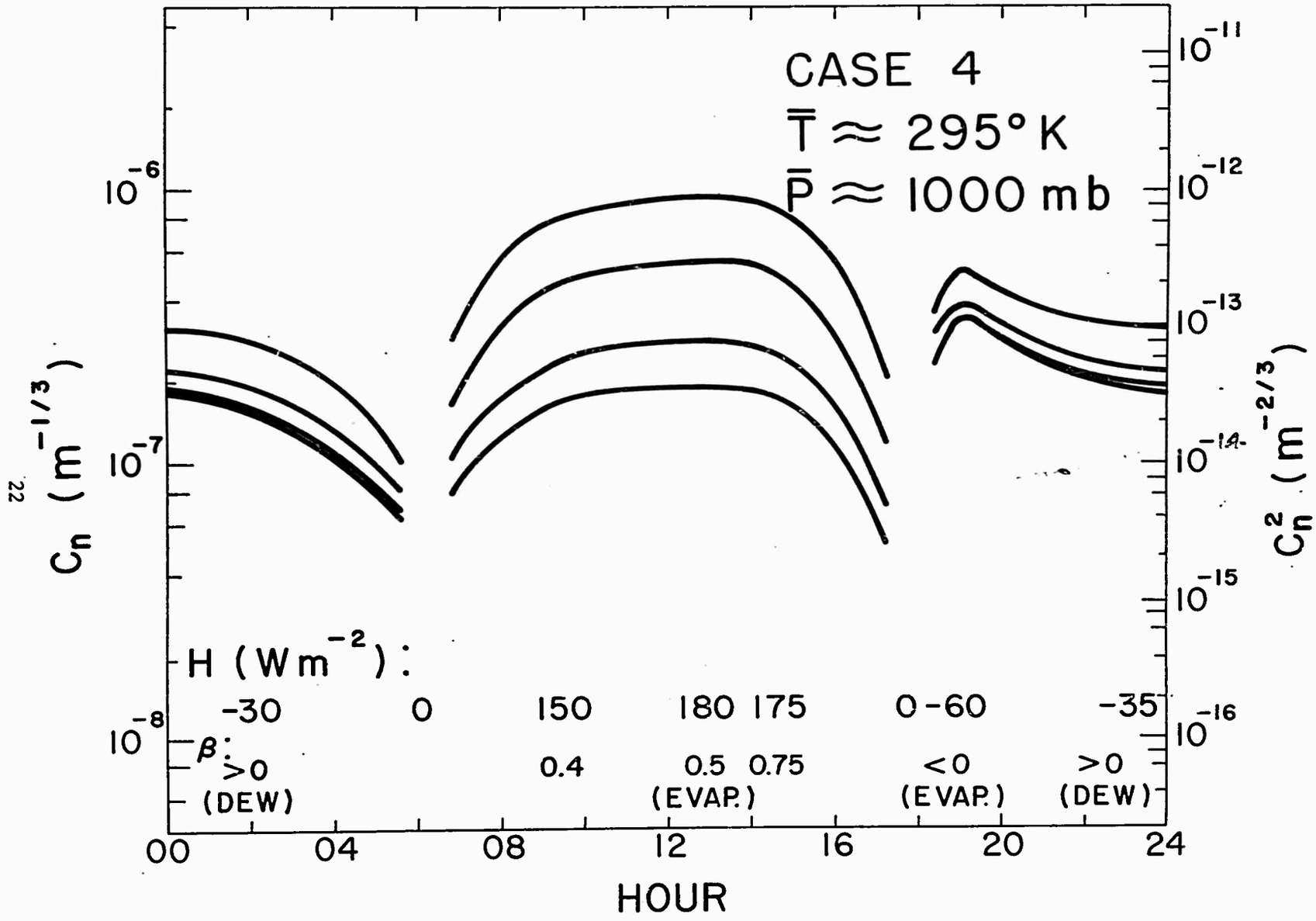


Figure 6 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 4.

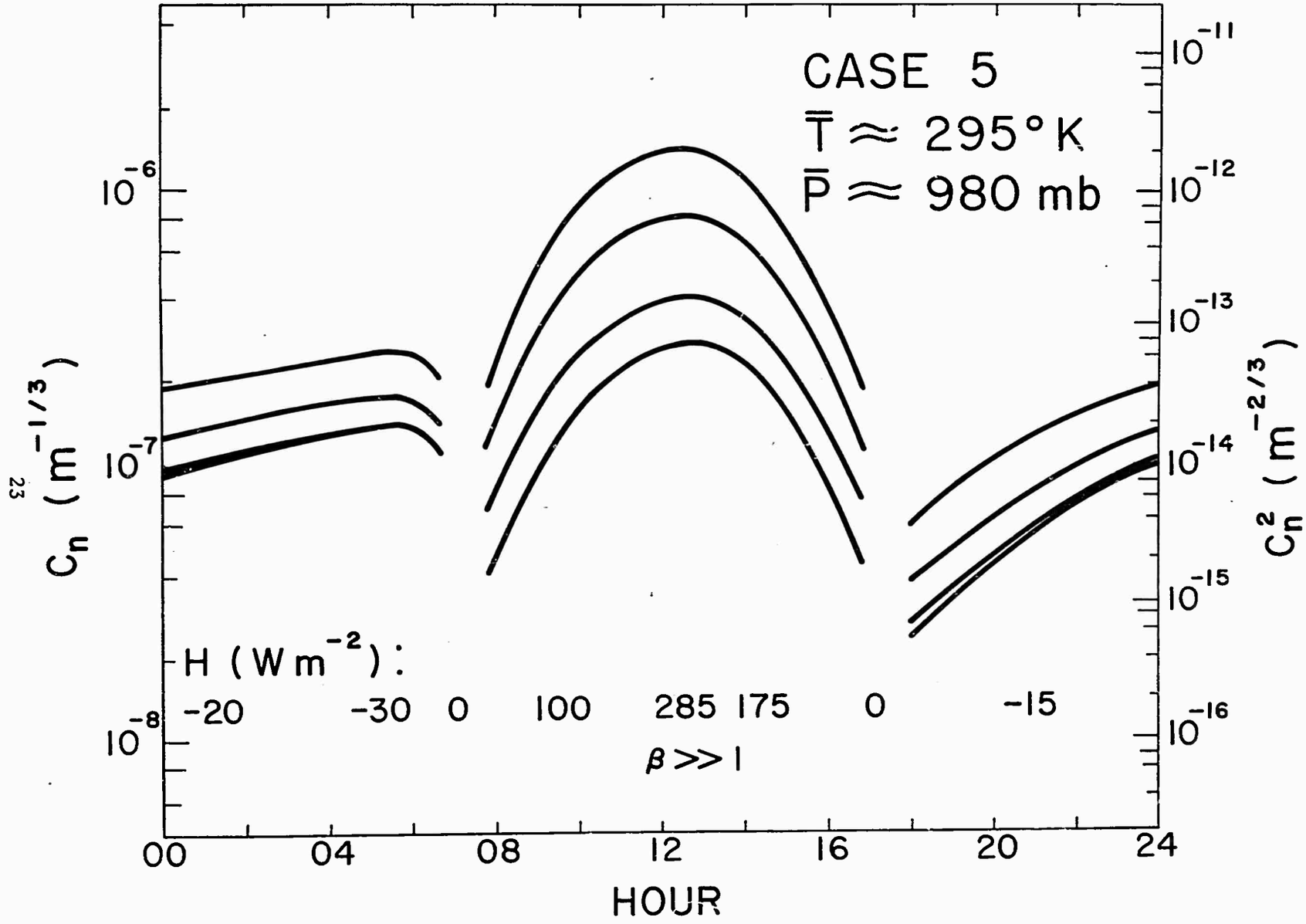


Figure 7 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 5.

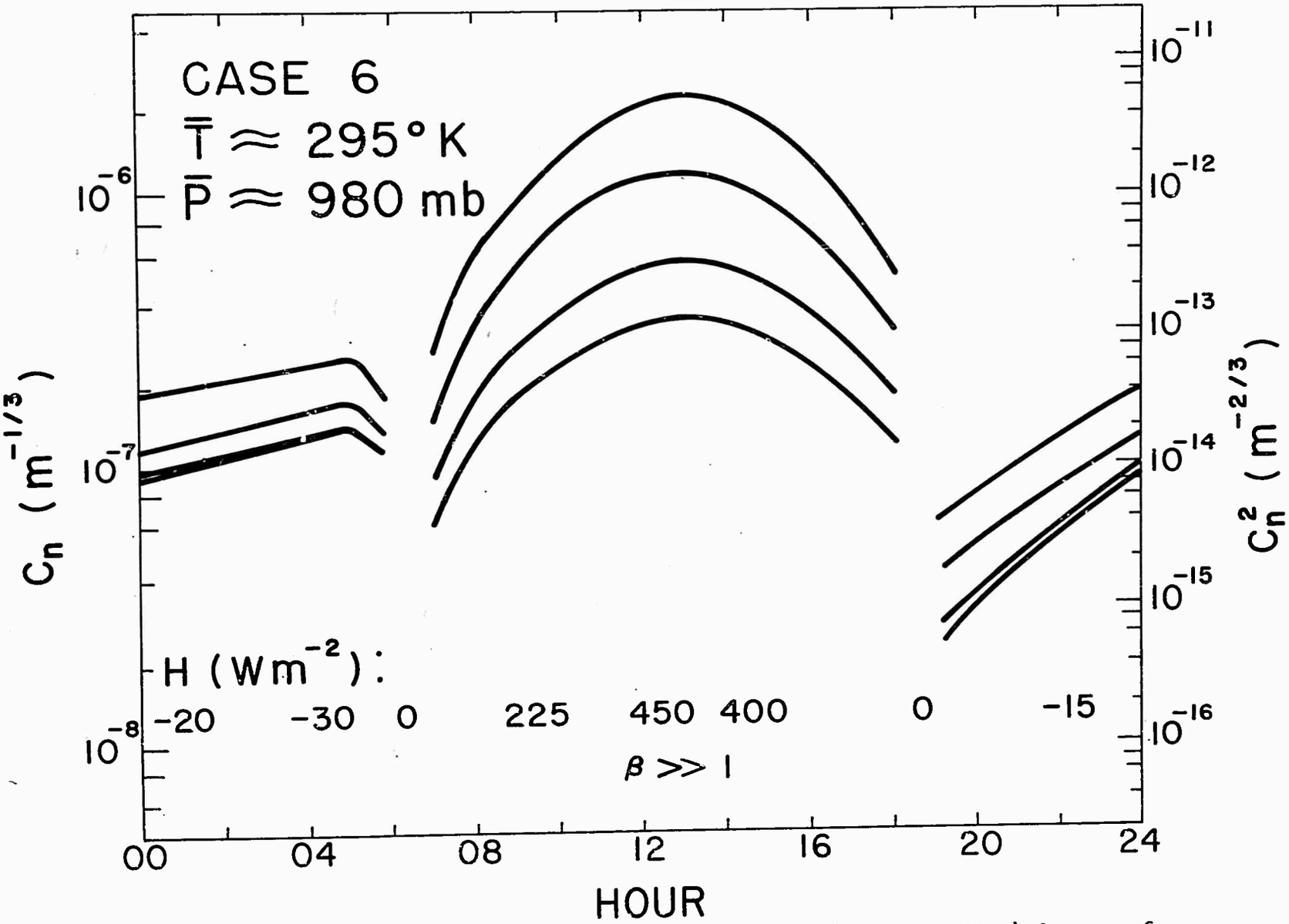


Figure 8 - $C_n$ vs mean solar time at 0.5, 2, 8, and 16 meters (top to bottom) for case 6.

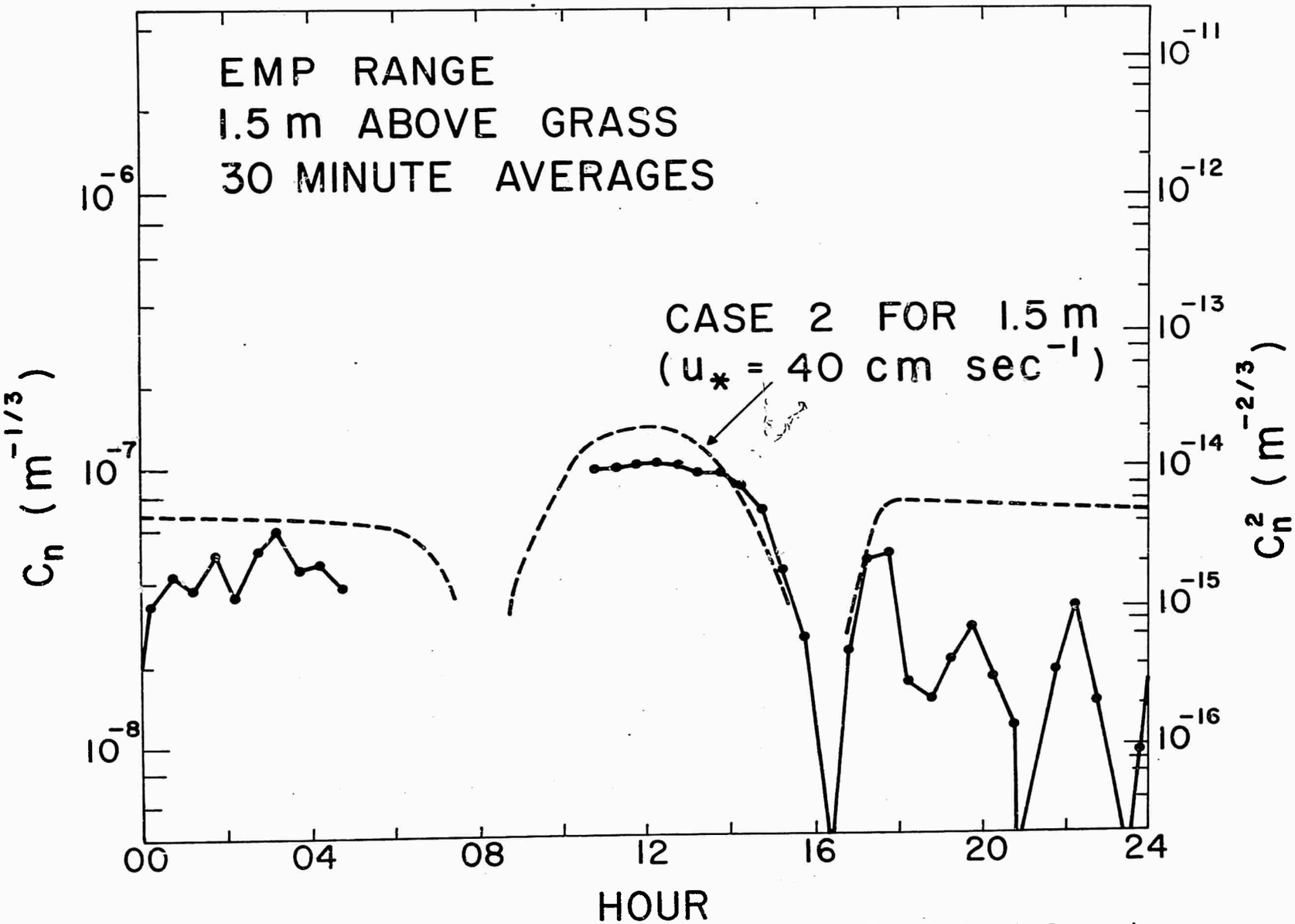


Figure 9 - Test of case 2 with measurements made at the U. S. Army Ballistic Research Laboratories, January 29 and 30, 1972.

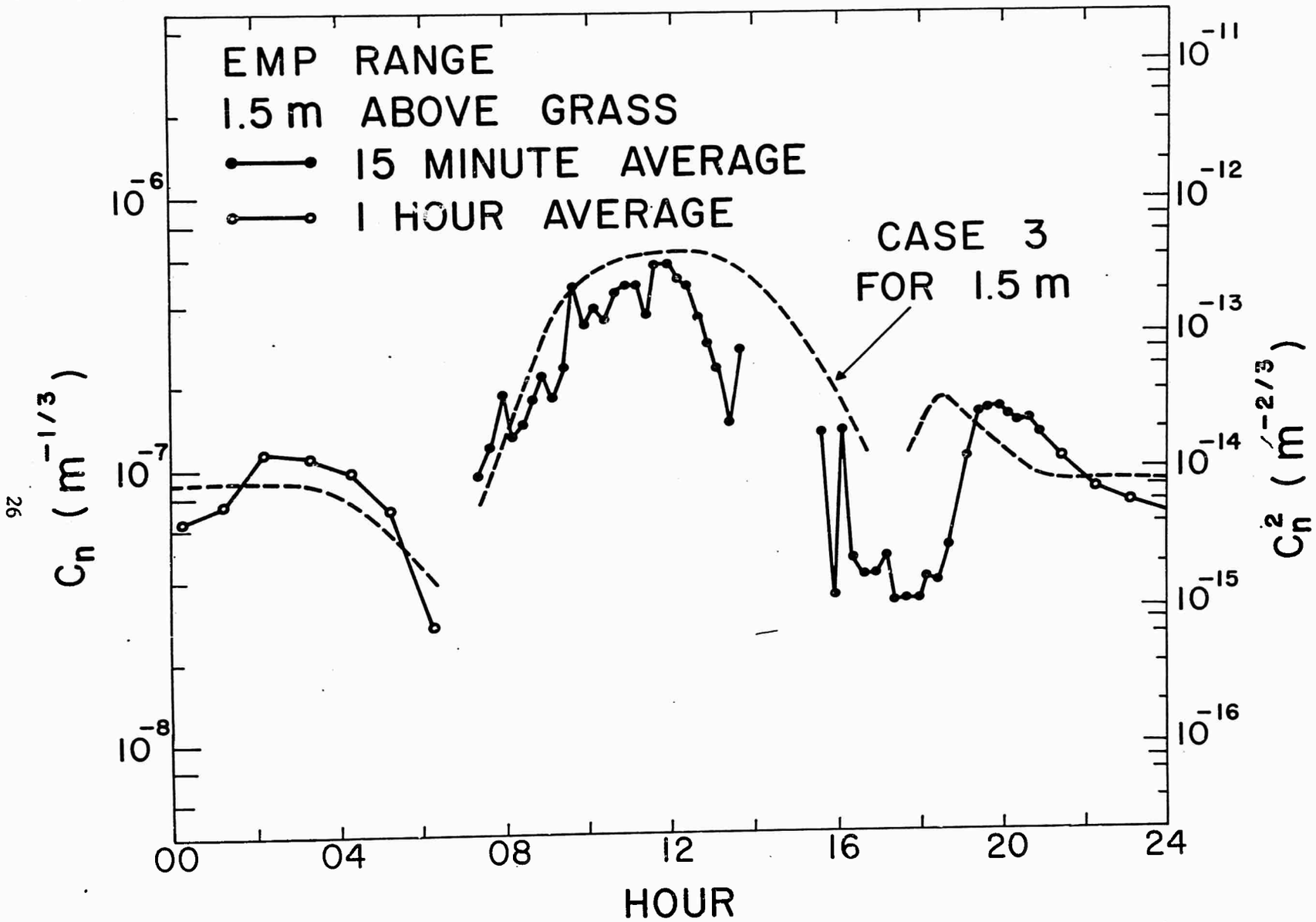


Figure 10 - Test of case 3 with measurements made at the U. S. Army Ballistic Research Laboratories, May 27 and 28, 1971.

Winter trends of $C_n$ are shown in Figs. 3 and 4, for which all air and surface temperatures are assumed below freezing. The sensible heat fluxes and thus $C_n$'s would be different above a thawing surface. Since the values of H in Figs. 3 and 4 only partially come from direct flux measurements, the values of $C_n$ are the least reliable of the six cases presented here. Nighttime H over snow was estimated using Neiderdorfer's [*Geiger*, 1961] nocturnal mean of about 17 W $m^{-2}$. Also, temperature lapse rates measured by *Geiger* [1961] above snow indicate that -H and thus $C_n$ gradually decrease during a clear night. Since during the daytime only a a slight decrease of temperature with height above snow was measured, small estimates of H were used, and they were chosen to be in phase with the expected solar radiation. The magnitudes of H at night in Fig. 4 are smaller than for Fig. 3 because of the lessened emitted radiative flux and increased soil heat flux for the grassy surface. For unstable conditions in Fig. 4, H is greater than in Fig. 3 because of the increased absorption of solar radiation.

As shown in Fig. 9, measurements were taken to test the curves in Fig. 4. The measurements were made above a level, brown grass surface on the Electromagnetic Propagation Range (EMP Range) at the U. S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland. The curves in Fig. 4 are shown to be reliable in Fig. 9, despite the low values of $C_n$ between 1700 and 2400 hours caused by extremely low winds, a common but sometimes unpredictable occurrence.

In Fig. 5, $C_n$ trends are shown for a wet, grassy surface in late spring or late summer, or in midsummer with a haze. The values of H chosen are typical of Central Europe [*Geiger*, 1961] and were obtained from measurements taken near Hancock, Wisconsin, by *Wesely et al.* [1970]. In Fig. 10 measurements of $C_n$ above grass at the EMP Range, Aberdeen Proving Ground, Maryland, show that the predictions are accurate. Clouds caused low $C_n$'s from 1600 to 1800 hours. Predictions of $C_n$ for a grassy, partially dry surface are shown in Fig. 6, where the values for H were chosen from Wesely's measurements near Davis, California. The H and

thus $C_n$ estimates are typical for the Great Plains of the United States [*Wyngaard et al.*, 1971; *Lettau and Davidson*, 1971] or the Russian Steppes [*Kallistratova and Timanovskiy*, 1971; *Zilitinkevich and Chalikov*, 1968] provided the soil is wet beneath the top few centimeters and the grass is actively growing. The maximum nighttime $C_n$'s for both Figs. 5 and 6 occur an hour or two after sunset because evaporation continues to cool the surface.

Figs. 7 and 8 are extremely dry climates in the winter and summer, respectively. However, the values of H and $C_n$ above grassy plains or steppes during a summer drought could approach those in Fig. 7 as the soil dries out and the grass turns brown. The values of H in Fig. 7 are taken directly from measurements made in Pampa de La Joya, Peru, by *Stearns* [1969]. Daytime values of H in Fig. 8 were taken from *Vehrencamp's* [1953] measurements at El Mirage, California, but since his nighttime values of wind speed and H were low, the nighttime values of H and $C_n$ in Fig. 8 were taken from Fig. 7. The predicted $C_n$'s of Fig. 8 have been partially substantiated by measurements made by T. H. Pries at the U. S. Army Atmospheric Sciences Laboratory at White Sands Missile Range where preliminary evaluation of data collected in early 1972 show peaks near $C_n = 7 \times 10^{-7} m^{-1/3}$ at a 3-meter height.

The trends drawn in Figs. 3 through 8 are poor climatological models because several assumptions of "ideal" conditions are made. Large reductions in $C_n$ occur whenever clouds obscure the sun or a significant portion of the sky at night. Reductions in visibility also lower the radiation absorbed at the surface and thus the $C_n$. Real diurnal curves are rarely as smooth as these "ideal" ensemble averages since wiggles, usually caused by nonstationary winds and other synoptic effects, can be expected. Equations 9 and 10 are valid only when the site is flat and homogeneous upwind from the measurement site. Estimates of the necessary fetch have been determined by several authors [*Elliot*, 1958; *Brooks*, 1961; *Panofsky and Townsend*, 1964; *Taylor*, 1969] to be from 50 to 100 times the height of the sample. If intermittency is strong, which

is most likely to occur at heights greater than 4 meters, then equation 13 is invalid and $C_n$ will be overestimated by equations 9 or 10. Finally discontinuities of $C_n$ do not exist as shown on the graphs for the neutral conditions near sunrise and sunset. The curves should approach zero, but $C_n$ is usually finite because the "fleeting moment" at which there are no temperature fluctuations is included in a ten-to-thirty-minute average essential for point measurements in the atmospheric surface layer. The curves for different heights should be slightly out of phase because of *Swinbank's* [1964] observation that the moment at which the temperature fluctuations go to zero at the surface may be as much as 15 minutes sooner than at 16 meters.

## CONCLUSION

The diurnal cycle predictions of $C_n$ presented in this paper indicate the magnitude of refractive index fluctuations in the inertial subrange of the atmospheric surface layer over land. These values of $C_n$ can be used in the various theoretical expressions that give the magnitude of amplitude and phase alterations of electromagnetic radiation through a thermally turbulent atmosphere. Low intermittency is assumed, and thus the results are most applicable to heights less than 4 meters. For a given climate, season and surface type the values of $C_n$ given here are the maximum expected since cloudless skies, excellent visibility and nearly optimum wind are assumed. Thus the diurnal cycles in Figs. 3 through 8 should be used only as rough approximations for clear skies, and during propagation experiments measurements of $C_n$ should be made. The simple, direct, structure function approach of equation 1 is more desirable than the indirect methods using equations 9 and 10, which rely upon semiempirical relationships. Regardless of the measurement technique chosen, one measurement usually suffices to determine $C_n$ at any height, since as shown by equation 9, $C_n$ is proportional to $z^{-1/3}$ in the atmospheric surface layer.

Future work in determining the diurnal behavior of $C_n$ in the atmospheric surface should include a study of the influence of water vapor fluctuations above oceans, lakes, and swamps. Recent flux measurements by *Pond et al.* [1971] above the sea show $\beta \sim 0.1$ within 10° of the equator, and according to equation 10 an estimate of $C_n$ based upon a direct measurement of $C_T$ would be too large by 30%. A smaller effect is expected at greater latitudes since seasonally averaged values of $\beta$ increase to nearly 0.5 at 70°N [*Sellers*, 1965]. The value of $\beta$ above oceans near shores and above small bodies of water can be exceptionally small and is usually negative during the daytime if the nearby land regions are dry [*Geiger*, 1961; *Sellers*, 1965]. Thus, at selected times and places water vapor fluctuations can greatly influence $C_n$.

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